

Terzaghi (1931)

## Contributed Articles

### THE STATIC RIGIDITY OF PLASTIC CLAYS

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According to Atterberg's definition<sup>1</sup> a clay can be considered to be in a plastic state, if its water content ranges between the lower and the upper limit of the plastic state. Although, by necessity, the methods for determining the limits are somewhat arbitrary, Atterberg's range of plasticity includes clay-water mixtures which have a fairly well-defined group of properties in common. Within this range an appreciable force is required to produce a continuous, plastic flow of the material and under the influence of forces beneath the "yield point" the material exhibits all of the characteristic elastic properties of imperfectly elastic solid bodies.<sup>2</sup>

The term "static rigidity" indicates the shearing stress "required just to cause the paste to flow and is a measure of the solid cohesive properties of the system."<sup>3</sup> In the stress-flow diagram the "static rigidity" is obtained by extrapolating the straight-line portion of the flow-line to the stress-axis. The static rigidity obviously corresponds to the term " $f$ " in Bingham's formula<sup>4</sup>  $dv = \mu(F-f)dr$ , wherein  $\mu$  denotes the "coefficient of mobility" and  $F$  the total stress.

In connection with the term "static rigidity" the following two questions arise: Is the "static rigidity" of a clay-water mixture determined by the water content of the paste or is it possible that the same clay containing the same quantity of water may have different static rigidities, depending on the preceding history of the sample? And does the static rigidity determine the lower limit of the stresses which produce continuous flow or is it possible that a continuous deformation may also be produced by stresses appreciably beneath the "static rigidity?"

#### Effect of the History of the Samples on the Static Rigidity

According to the definition quoted above, the "static resistance" in every respect corresponds to the "shearing strength" of plastic materials, *i. e.*, to the maximum shearing stress such materials can sustain without becoming subject to continuous deformation. The ratio between the shearing strength and the compressive strength of unconfined samples has, for any given material, a fairly constant value. Hence the conclusions derived from compression tests with unconfined samples are also valid for the "static resistance." During the last few years, the following facts concerning the compressive strength have been found:

(a) Two identical clays with identical water-contents can have very different compressive strength depending upon whether or not the water contained in the voids of the clay is in a state of tension. This fact was found by means of the following experiment:<sup>5</sup> All the tests were started with samples with a water-content  $w_1$ . One set of the samples was consolidated under pressure into different states represented by the points of the curve AC of Figure 1. Then both the pressure and the excess water were removed rapidly enough to prevent the samples from swelling, and the compressive strength was determined. The samples of the second set were consolidated under somewhat higher pressures than those to which the first set were subjected, and after consolidation they were allowed to swell in contact with water under no pressure and the compressive strength was determined. Thus, by consolidating a sample D of the second set

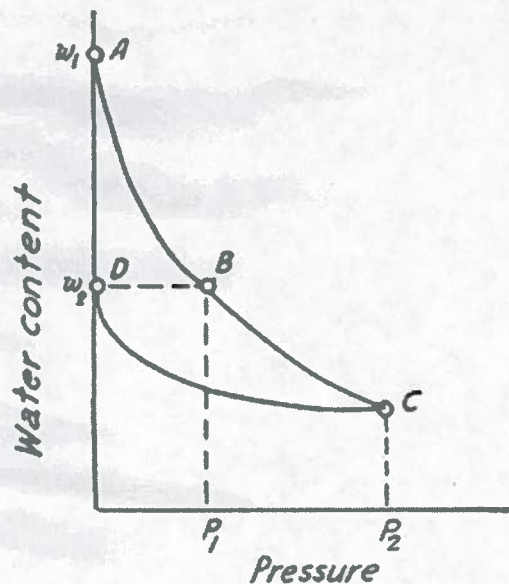


FIGURE 1

under a high pressure  $p_2$  (Figure 1) and subsequently allowing it to swell, it was possible to arrive at a water-content  $w_2$  equal to the water-content  $w_2$  of a sample B, of the first set, consolidated under smaller pressure  $p_1$  without subsequent swelling. By comparing the results of the tests made upon the two sets of samples it was found that both the modulus of elasticity and the compressive strength of any sample D of the second set amounted to less than one-half of the corresponding values for the samples B of the first set with equal water content.

(b) If a sample of clay is kneaded and then allowed to rest without

loss of water through evaporation (being kept in a humidifier) its compressive strength may gradually increase during a period of several days. This phenomenon seems to correspond to the "thixotropic" effects characteristic of certain colloidal solutions.<sup>9</sup>

(c) Kneading a sample of "undisturbed" clay, cut out of a natural clay deposit almost invariably reduced both its compressive strength and its modulus of elasticity to a fraction of the corresponding values for the undisturbed sample.<sup>6</sup> The value of this fraction may range between the limits of  $1/2$  and  $1/50$ .

(d) Observations concerning the settlement of buildings resting on thick clay deposits lead one to suspect that the rate of plastic flow may spontaneously increase several months or even many years after it started.

$I \rightarrow D > D_{max}$

These observations make it apparent that the "static rigidity" of plastic clays depends on many factors other than the water-content and the nature of the solid phase.

### Plastic Flow under Stress beneath the Yield Point

If an unconfined sample of soft plastic clay is submitted to a compressive stress above the yield point, the following movements take place: A rapid elastic and inelastic yield corresponding to the rapid yield of a solid under stress, then a slow elastic and inelastic yield corresponding to the elastic after-effect of solids and finally a flow with constant speed. Each one of these movements has characteristics and physical causes different from those of the others. However, at least at the outset, all these causes and with them the corresponding movements simultaneously exist. As the process goes on, the movements die out one by one, until the pure flow remains. Figure 2 shows graphically the relation between time and movement. For the strain effect of compressive stresses beneath the yield point we obtain perfectly analogous time-strain diagrams, except for the fact that the flow is extremely insignificant if compared with the preceding movements. Available empirical evidence plainly indicates that the relation between stress and plastic flow for plastic clays is similar to that shown in Figure 3.

According to this figure, the "static rigidity" is considerably greater than the shearing stress just required to cause the clay to flow. Hence distinction should be made between the "yield point" *B* and the "flow point" *A*. Thus for several buildings in New England it is known that they have been settling for years at an almost uniform rate of about 1" per year although the stresses produced by the weight of the structure in the underlying clay deposits are considerably below the "yield point." Since none of the other causes of settlement leads to a subsidence at a uniform rate we are inclined to attribute the movement to "flow."

According to the conceptions of the author,<sup>7</sup> the flow under stress beneath and above the "yield point" may be due to essentially different causes. Under a stress beneath the yield point the flow is supposed to be caused by viscous flow within the layers of adsorbed waters adjoining the points of contact between the soil grains. Within spaces with a width of less than  $100\mu\mu$  the viscosity of the water is very much higher than the normal viscosity.<sup>2</sup> Yet the water is still in a liquid state, not assuming

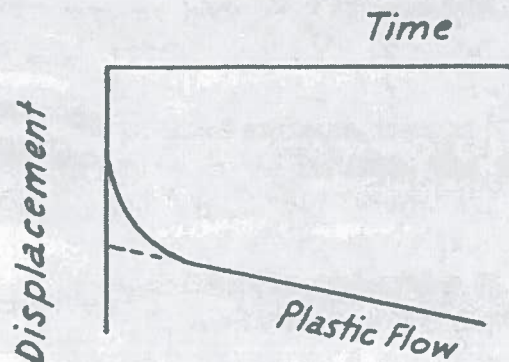


FIGURE 2

$$\mu = 10^{-6} \text{ m} = 10^{-3} \text{ mm}$$

$$\mu\mu = 10^{-12} \text{ m} = 10^{-9} \text{ mm}$$

$$100\mu\mu = 10^{-7} \text{ m} = 1 \text{ \AA}$$

the properties of a solid body unless the distance between the walls is very much smaller than  $100\mu\mu$ . This statement is corroborated by the results of tests concerning the adhesion between optically plane steel surfaces, performed by H. M. Budgett.<sup>8</sup> The distance between the faces was smaller than  $100\mu\mu$  and the space was filled with condensed water vapor. In the atmosphere the adhesion between the two surfaces was found to be equal to 3.5 atmospheres, while in the vacuum, the adhesion was 3.31 atmospheres. A solid film of water cannot possibly transmit the air pressure through hydrostatic pressure to the steel surface. Hence if the film of water had been solid over the entire area of the surface of contact,

the adhesion should, in the vacuum, have been by 1 atmosphere smaller than in the atmosphere, viz., equal to 2.5 atmospheres. The fact that the adhesion in the vacuum was as high as 3.31 atmospheres demonstrates that the film had, in spite of the small distance between the polished surfaces, over at least 80 per cent of its area, the properties of a liquid (Figure 4).

In a mass of clay every point of contact between the grains (*i. e.*, every spot where the soil grains are bound together by a solid film) is surrounded by a film which, in spite of its very high viscosity, still has the properties of a liquid. Under the influence of a stress with increasing intensity, the flow point (A, Figure 3) is reached as soon as the stress becomes equal to the bond produced by the solid

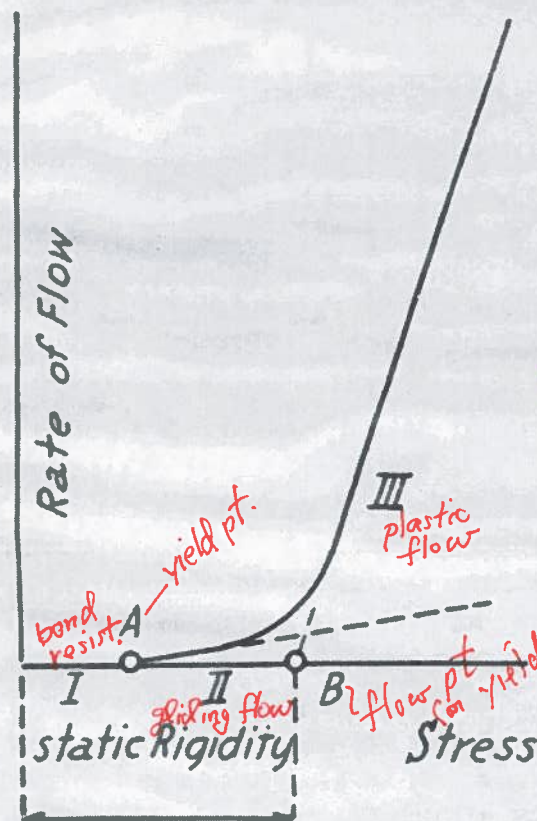


FIGURE 3

part of the film. Further increase of the stress produces a slow viscous flow in the adjoining liquid parts of the film. However, due to the high viscosity of this part the flow is very slow. It seems to correspond to what E. C. Bingham calls the "slippage."<sup>4</sup> Yet, according to the conclusions of the writer, the movement is not due to "lack of sufficient adhesion between the material and the shearing surface." It merely results from the fact that part of the cementing films have the properties of liquids.

With increasing intensity of the stress, the sliding velocity increases and with the velocity also increases the viscous resistance until the viscous resistance becomes greater than the adhesion between the films and the

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solid. The film tears off and this event corresponds to the "yield point." Hence, according to this conception, the "static rigidity" is equal to the sum of the adhesion between the soil grains and both the solid and the adjoining highly viscous parts of the liquid films. If this conception holds true, the term "static rigidity" must be considered misleading, because it involves the statement that no flow should take place under stresses which are smaller. Hence it is proposed to eliminate this term for the stress corresponding to the yield point *B*, Figure 3, and to replace it by the term

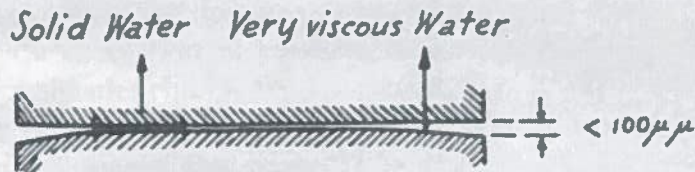


FIGURE 4

"shearing resistance," because it is the minimum stress required to produce actual shear. The stress corresponding to point *A*, Figure 3, may be called the "bond resistance" because it solely represents the shearing resistance of the solid part of the cementing water films. The flow which takes place between points *A* and *B* (range II) may be termed the "gliding flow" in contrast to the plastic flow, characteristic for the stress range III.<sup>10</sup>

Under a stress above the yield point (range III, Figure 3) the clay undergoes a continuous deformation, called the "plastic flow." This process seems to consist of successively establishing and breaking the bond between adjoining particles whereby the particles change their partners. Figure 5 may serve to illustrate this conception by means of a mechanical analogy. Two rough surfaces are supposed to adhere at the points  $a_1, a_2, a_3$ , etc., with a force of  $f$  per unit of total area (shearing resistance). Adhesion is supposed to be produced by the cementing action of a film similar to the film shown in Figure 4. The film is partly solid, partly liquid. Under feeble stress no movement occurs. At a stress above the "flow point" (*A*, Figure 3) the resistance of the solid part of the film is insufficient to prevent slow viscous flow within the liquid part. At the "yield point" the contact which existed at the points  $a_1, a_2, a_3$ , etc., breaks and the two surfaces slide over each other. However, as soon as  $a_1'$  arrives at the place of  $a_2$ , the two surfaces "catch" again and work must be performed to break the newly established bond. The force available for performing this work is

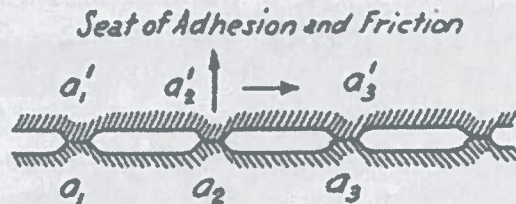


FIGURE 5

determined by the difference ( $F-f$ ) between the total stress  $F$  and the shearing resistance  $f$ . The greater this difference, the more often the task of breaking a new contact can be performed per unit of time. For this reason, the speed of the plastic flow increases in simple proportion to the difference ( $F-f$ ) (straight line  $BC$ , Figure 3).

It is evident that the analogy represented by Figure 5 is a very crude one because it neglects the resistances, viscous or otherwise, which develop on the path while a point  $a_1'$  travels from  $a_1$  to  $a_2$ . Yet it illustrates the essence of the writer's conception and it is tempting to explain the plastic flow of plastic solids, for instance lead, in a similar manner; that is, to consider it as a succession of breaking and establishing molecular bonds, whereby the molecules change their partners.

It remains to consider whether or not the phenomena indicated by the rise of the curve within the range II in Figure 3 and those responsible for the corresponding curve of transition in the diagrams derived from the capillary method are identical. The writer admits the existence of a certain relationship, but he denies the identity. In both cases the resistance against movement resides in the viscosity of a film of water. However the rise of the curve in Figure 3 expresses a flow within the clay, while the corresponding rise in the "tube diagrams" is due to the sliding between the tube and the plug of clay. For this reason the position of the flow-point in the tube diagram is by no means necessarily identical with the point called "flow-point" in the diagram Figure 3. The tube method is not sensitive enough to disclose movements as delicate as those which occur between the grains prior to the plastic flow.

### The Pore-Space for Zero-Mobility

According to E. C. Bingham,<sup>4</sup> the pore-space for zero-mobility is equal to roughly 50 per cent. In an accumulation of grains with a smaller pore-space "the particles are interlocked and no true flow is possible, but rupture, with disintegration of the particles." For accumulations with bulky grains this conception is obviously correct. However, for very fat, greasy clays and certain fine muds the pore-space for zero-mobility seems to be considerably lower, 40 per cent or less. In plastic clays whose pore-space is below zero-mobility, only the gliding flow (range II, Figure 3) seems to occur. Furthermore, for clays the pore-space for zero-mobility is even less definite than it is for sands, and it is doubtful whether one is entitled to attribute to the pore-space for zero-mobility a value considerably above Atterberg's "lower limit of the plastic state."

### Conclusions

(1) Two identical clays with identical water contents may have very different "static resistance" depending on the history of the samples,

Since the capillary tube method involves experimenting with thoroughly kneaded samples, the values obtained by means of this method have no validity for clays in the natural state.

(2) When a sample of plastic clay is submitted to a stress with increasing intensity, the plastic flow is preceded by a gliding flow, which seems to have its seat in the highly viscous water surrounding the points of actual adhesion. The stress at which this "gliding flow" starts is considerably smaller than the "static resistance." For this reason it is believed that the "static resistance" represents the sum of the adhesion between the solid particles and both the solid and the highly viscous films of water. The threshold value at which the gliding flow starts was called the "bond resistance" because it represents the shearing strength solely of the solid part of the films of water. The term "static rigidity" is misleading because it involves the conception that no flow takes place under stresses which are smaller. It is proposed to call it "shearing resistance."

The "slippage" which occurs in the "capillary-tube test" under stress below the yield-point is due to causes somewhat different from those of the "gliding flow" within the clay. Hence the position of the "flow-point" cannot be determined by means of the capillary tube method.

(3) A stress below the bond resistance produces a rapid elastic and inelastic yield followed by a gradual elastic and inelastic yield. A stress between the bond- and the shearing-resistance produces a gliding flow in addition to the four motions mentioned before. A stress above the shearing resistance produces in addition a plastic flow. At the very instant of stress application, all these motions simultaneously exist. As time goes on the first two motions, then the second two motions die out, and only the gliding and the plastic flow continue.

(4) The values furnished by shearing tests and the values computed from the results of compression tests represent the shearing resistance, while the stability of foundations depends on keeping the stresses below the "bond resistance." For this reason a closer investigation of the "gliding flow" preceding the plastic flow is of outstanding practical importance.

(5) It is suggested that the plastic flow consists in alternately breaking and reestablishing the bond between the particles, whereby the particles change their partners. The force available for performing this work is equal to the difference between the total stress and the shearing resistance. The excess force required for maintaining the flow increases in simple proportion with the number of "contacts" per unit of time which in turn increases in simple proportion with the speed of the flow.

### References

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- <sup>2</sup> K. Terzaghi, *Erdbaumechanik* (1925).

<sup>3</sup> B. A. Keen and G. W. Scott Blair, "Plastometric Studies of Soil and Clay Pastes," *J. Agr. Sci.*, 19, Part IV, 684 (1929).

<sup>4</sup> E. C. Bingham, *Fluidity and Plasticity*, 1st ed. (1922).

<sup>5</sup> K. Terzaghi, *Mechanik der Schuettungen, Sedimente und Gele* in Auerbach und Hort, *Handbuch der physikalischen und technischen Mechanik*, IV, 2, Berlin (1931).

<sup>6</sup> K. Terzaghi, "Determination of the Consistency of Soils by Means of Penetration Tests," *Public Roads*, 7 (1927).

<sup>7</sup> K. Terzaghi, "Die Tragfaehigkeit von Pfahlgruendungen," *Die Bautechnik* (1930).

<sup>8</sup> *Proc. Roy. Soc. (London)*, 86A, 25 (1912).

<sup>9</sup> H. Freundlich, "Über Thixotropie," *Kolloid. Z.* 46, 289 (1928).

<sup>10</sup> The "shearing resistance" reminds of W. B. Hardy's A-value for the adhesion between liquids and solids. (Studies in Adhesion I. *Proc. Roy. Soc. (London)*, 112A, 62 (1926). Yet these two values are not perfectly identical, for the following reason: Hardy's A-value determines the adhesion between a solid substance and a liquid irrespective of the degree of viscosity of the film, while the "shearing resistance" merely includes the adhesion between the solid substance and the most viscous part of the liquid films, contained in the clay. The other portions of the films do not participate in producing "shearing resistance." This distinction involves a rather sharp boundary between the normal and the highly viscous part of the films. However, according to the investigations of the writer concerning the viscosity of the water in narrow openings, such a fairly sharp boundary should exist. (Terzaghi, *Erdbau-mechanik* (1925).

VIENNA,

December 4, 1930

### Discussion

MISS COBB: It is interesting to see that the New England soil is slipping out from the Custom House Tower for the same reason that coatings slip off paper—*i. e.*, due to the insidiously viscous nature of the vehicle. The point about the rigidity of the film of water is very important. I wonder if some one would give a theoretical explanation of the high viscosity of a thin layer?

MR. MILLER: The best thing that can be said in this connection is to quote Dr. Terzaghi's paper in the Third Colloid Symposium Monograph. He considers such thin films as, "zones of forced vibrations" in which the molecules of water wetting the solid take on its much smaller molecular vibrations. The result of this decreased molecular vibration is identical to undercooling with its concomitant great rise in viscosity. I think he has chosen a highly suggestive phrase in describing the absorbed film as a "zone of forced vibrations."

MR. HERSHEL: It is not quite clear to me why they should take on these vibrations. Why should they? It does not explain it to my mind at all. (Laughter.)

MR. MILLER: As I interpret Dr. Terzaghi, when wetting occurs there is an attraction of the solid for the liquid particles, with a force greater than that of their kinetic activity. They now lose their vibrations of fairly large amplitude and take on the much smaller vibrations of the solid and the water acts like an under-cooled liquid.

CHAIRMAN DAVEY: You mean that that sticks to the next layer of water and so on, so that there is a shading off?

MR. MILLER: Each layer of water molecules with suppressed kinetic activity suppresses to some extent the next so we have as Terzaghi said "a zone of forced vibrations."

DR. STEELE: I should assume then that we have water acting simply as a solid and reduce its vibrations to the vibrations of a solid.



MR. MILLER: Yes, in the first layer or two of molecules in cases of strong absorption and shading off more or less rapidly depending on the temperature.

CHAIRMAN DAVEY: We would like to think of an adsorbed layer which progressively becomes more and more disorganized as we go away from the adsorbing material in the body of the liquid. Experimentally, from the x-ray standpoint, there is no evidence of such a graded layer. We might, I suppose, account for this lack of evidence by saying that the layer next to the adsorbing material is truly oriented; that the next layer is composed of molecules which are tied at one end to the adsorbed layer, but which have considerable freedom of motion due to the effect of temperature. This gives us, then, a partial orientation which is far from being complete enough to show x-ray diffraction.

MR. HERSCHEL: Will you kindly sketch on the board the orientation of molecules adjacent to a metal surface? (Laughter.) I did not quite follow your description and I would like to see just what it looks like.

CHAIRMAN DAVEY: I cannot tell what it looks like; I can tell you what I think it looks like.

MR. HERSCHEL: That is all I want. (Laughter.)

CHAIRMAN DAVEY: This is the surface of the solid on which we have a layer just one molecule thick. If now we have the next layer tilted at various angles we can still have more of a component upward than we have in either of those directions parallel to the adsorbing surface. In other words, we could have an attempt at orientation but no actual orientation, and that attempt should be more and more feeble the further out you go. Of course this is only theory; it does not have any experimental foundation in x-ray work. All we can say is that there is no experimental evidence which is contrary to this theory.

MR. HERSCHEL: Then I understand that you are not convinced that there is any evidence in support of this phenomenon known as flow orientation; that is, orientation in addition to the laminar layers due to pressure or slip, which is only brought about by the primary orientation due to attractive forces of the molecules?

CHAIRMAN DAVEY: No, I would not say there is no evidence; I would say there is no x-ray evidence, and the x-ray evidence, to my mind, is the thing which is the final criterion. If we could get x-ray evidence that there was progressive orientation, we could say that it is actually there, but if we cannot get the x-ray evidence, all we can say is that we have an hypothesis.

MR. HERSCHEL: Can't you be convinced by any other evidence except x-ray evidence?

CHAIRMAN DAVEY: No, because the actual alignment of molecules would show by the x-ray evidence; partial alignment would not.

DR. STEELE: The question of rate at which orientation shades off from the interface out into the liquid is one that requires even more attention than it has received in the past, if that be possible. I have been moved to make a few remarks on this question several times during the meeting, and now I would like to make them, even though they may not all apply directly to this paper.

In the first place, it has been mentioned several times that Trillat has studied oriented layers many molecules deep of fatty acids on solids and liquids by means of x-rays. While some one of his papers may have escaped my attention, I have tried to keep informed on his work, and, to my knowledge, he has not worked above the melting point of the fatty acid in any case. In other words, he showed that when a fatty acid crystallized on a surface, mercury, glass, or what not, the resulting orientation was always the same. I believe that his data offer no evidence as to the thickness of adsorbed layers of liquids at interfaces.

In our laboratory we have made many attempts to obtain such diffraction from

fatty acids above their melting point without success, although we had no difficulty in obtaining the diffraction at lower temperatures. This is another bit of negative evidence on the persistence of orientation of fatty acid many molecules away from an interface. There is, however, considerable evidence pointing to oriented layers many molecules thick; yet the whole problem is far from solved.

To my mind the importance of the study of the effect of solids on the liquids in which they are dispersed can be hardly over-emphasized. Let us take, for example, the old experiment with kerosene and zinc oxide. They are mixed together to form a stiff paste which has a very high yield value and is capable of taking not more than a very small elastic deformation. When a few drops of a dispersing agent, such as a fatty acid, are added, the mixture will run off the mixing glass. The yield value is too small to be observed by any ordinary method. The explanation of these phenomena is familiar to every one. The zinc oxide was originally flocculated giving a high yield value. The dispersing agent concentrated at the interface reduced the interfacial tension and destroyed the flocculation with its attendant solid-to-solid friction and high yield value. This explanation is well established and generally accepted.

If, however, the zinc oxide be dispersed in rubber, the picture becomes less distinct. It increases the yield value of the raw rubber, makes it much stiffer, and it has been commonly supposed that this also is due to flocculation. In addition, however, it increases the elasticity of the mix, while if the pigment were flocculated the mix should be inelastic and "dead." It seems, at times, as though the zinc oxide had caused slight vulcanization. Is it possible that the rubber molecules are so tightly adsorbed, so thoroughly oriented, that a sort of pseudo-vulcanization has actually resulted and that the inherent elasticity of the molecules has come into play? Other systems, such as elastic bentonite gels, offer somewhat the same problem. The dimensions of the adsorbed water layers on bentonite which Hauser has postulated seem quite out of reason, but no other satisfactory explanation of the facts has been offered. There is probably no field in the science of Rheology more worthy of study than these solid-liquid relationships.