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Generalizing and Measuring the Hvorslev Effective Components of Shear Resistance

by

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GENERALIZING AND MEASURING THE HVORSLEV EFFECTIVE COMPONENTS OF SHEAR RESISTANCE

By John H. Schmertmann[1]

Synopsis

The Hvorslev effective components are reviewed, and it is argued that the usual assumption of equal cohesion at equal void ratio can be in error because void ratio is not a sufficient description of soil structure. To avoid this error, and to permit investigating the components as functions of strain, it is necessary to redefine and generalize them. This is done. An experimental procedure, called the IDS test, is described which permits separation of the generalized components in a direct manner from a single specimen. The test is versatile as well as practical. The few available comparisons between the Hvorslev and generalized components are presented and are favorable.

Hvorslev Components

Hvorslev (**1,2**)[2] presented and later reviewed his work in determining shear strength[3] components, which he called effective cohesion, c_e, and effective friction, φ_e. He formulated the results of his experiments with two remolded clays, and the resulting equation for each had the simple Coulomb form, thus defining c_e and φ_e. Because these components were based on strength behavior at different effective stress but equal void ratio on the failure plane at failure, and c_e at a given void ratio appeared independent of whether in the virgin, rebound, or recompression phase of stress history, they have also become com monly known as the "fundamental" o "true" components of soil strength.

The concept of these components ha endured with few corrections or addition since the work of Hvorslev. Bjerrun (**3**, p. 80) Bjerrum and Wu (**4**), anc Jakobson (**5**) have indicated that fo some undisturbed clays, or clays re molded at water contents below th liquid limit, there exists an "origi cohesion" to be added to the cohesio which is a function of only void ratio Gibson (**6**) showed that the component determined from drained tests must b corrected for volume-change effects, i any.

Because the determination of c_e anc φ_e is a laborious procedure often requir ing unobtainable duplicate specimens, i has been done in research for only ver few undisturbed soils, and perhaps neve for purely practical work. Yet the effec tive components have had great practica

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[2] The boldface numbers in parentheses refer to the list of references appended to this paper.

[3] The term shear "resistance" refers to the shear stress mobilized by the soil at any given condition, while shear "strength" refers only to the shear resistance at a defined failure condition.

significance. Their mere existence has been the rallying-point for the continued use of the component philosophy of soil behavior. Their use permitted the mathematical studies which are the theoretical bases for some common empirical methods of applying soil strength analysis in practice. Some examples are Skempton's (7) analysis of undrained strength and the $\varphi = 0$ analysis, Hansen and Gibson's (8) extension to the case of anisotropic consolidation and consideration of the comparisons between different types of tests for undrained strength, and Jakobson's(5) further extension to include origin cohesion and octahedral normal stress. Bjerrum (3, p. 79) and Skempton and Bishop (9) used the effective components to show the relationship between drained and undrained strength envelopes and between water content and undrained strength.

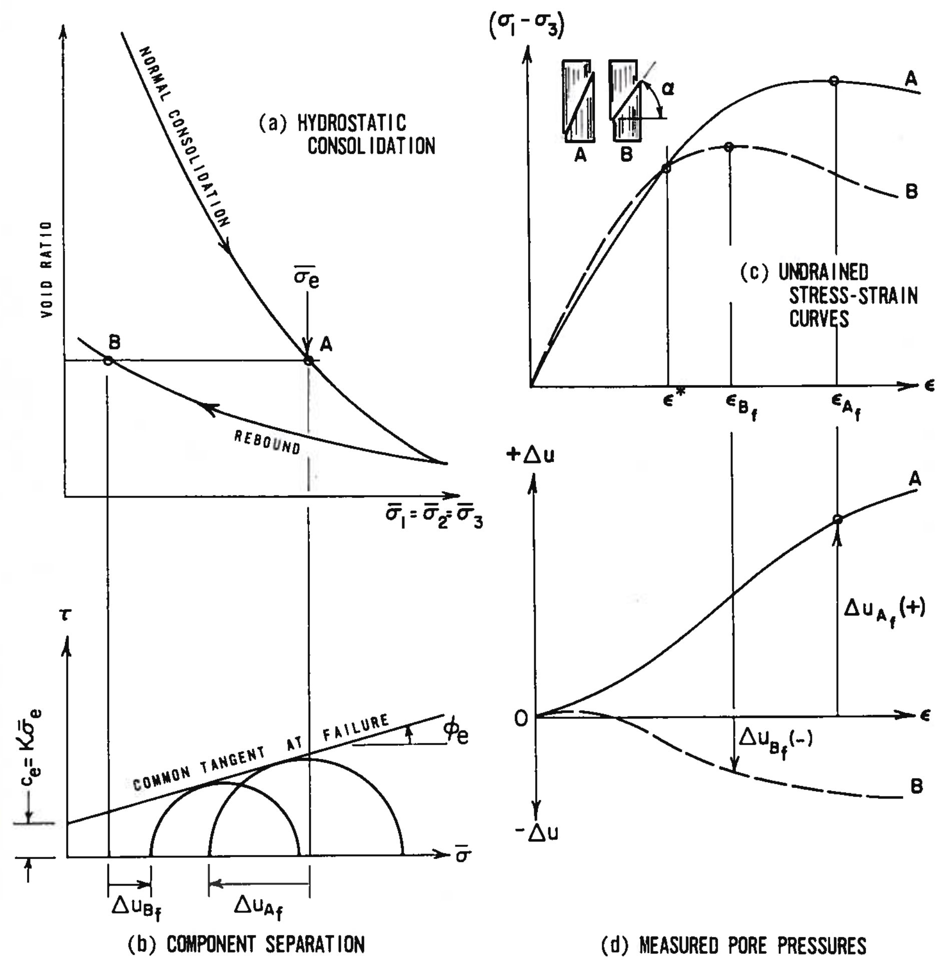


FIG. 1—Hydrostatically Consolidated, Undrained Triaxial Tests (Fictional) to Determine Effective Components and Illustrate Their Physical Meaning.

VOID RATIO AND STRUCTURE

Intended Physical Meaning of Effective Strength Components:

The well-known experimental method suggested by Terzaghi (10) to determine c_e and φ_e, which also is generally considered to illustrate excellently the physical meaning of the effective components, is reviewed in Fig. 1(*a*) and (*b*). This method and its relationship to four other methods is also discussed by Bjerrum (3). Following Hvorslev (1), the assumption is made that because tests *A* and *B* have the same void ratio at failure they have the same c_e, and all strength differences are due to $(\Delta\bar{\sigma})$ tan φ_e. Most investigators would agree that effective cohesion, because it is believed to be "fundamental," is some function of the noneffective stresses or bonds between particles and of the details of the extremely complex geometry of particle spacing and contacts. We can say that c_e is a function of the catch-all term "soil structure" or just "structure." Methods-of-testing will be considered a constant here, and possible dilatancy corrections, as suggested by Gibson, are also not considered for simplicity.

When assuming equal cohesion at equal void ratio it is therefore also assumed that equal void ratio means equal structure. Of course, Hvorslev realized this point. For example, he wrote: "Furthermore, it must also be assumed that there is no significant difference in the geometric structure of the test specimens in a given series at the time of failure" (2), p. 206.

The meaning intended for these components is then:

> These are the two components of a soil's shear strength in which one, effective friction, is completely and linearly responsible for the difference in shear strength between two specimens with the same structure but with different effective stress on the failure plane. The other, effective cohesion, is the strengh remaining after extrapolating to zero effective stress.

But this intended meaning may not be clearly achieved by an experimental determination using the Terzaghi method or a variation of it. Some of the following discussion refers to Fig. 1, in which is presented imaginary data from a simple 2-test series to determine the effective components by the Terzaghi method. For illustration, the author has imagined test data which involve and perhaps exaggerate several difficulties in evaluating the significance or accuracy of the components thus separated.

Two Factors Influencing Structure:

Void ratio is of course an important gross measure of soil structure. However, it tells us nothing about the important details of structure. For example, an undisturbed and a remolded sensitive clay can have the same void ratio, but structure is certainly very different. Even assuming void ratio is necessary to describe structure, the question is whether it is sufficient. The author argues that in general it is not, but in the special circumstances originally encountered by Hvorslev it could be sufficient.

Overconsolidation—Fig. 1(*c*) and (*d*) illustrate stress-strain and pore pressure-strain curves that might be expected from two consolidated-undrained triaxial tests such as *A* and *B*. The initial modulus is a measure of the "stiffness" of the soil structure. It could well be stiffer and therefore obviously different for test *B* in spite of lower effective stress. Pore pressure is a measure of the tendency of a saturated structure to reduce volume with strain. The structure of *B* wants to expand while that of *A* wants to contract. During consolidation it is necessary to imagine the occurrence of micro-shear failure, or slippages, at many contact points throughout the mass. These slippages permit the platey particles to assume a more stacked (dispersed) arrangement which is a necessary corequisite to the void ratio reduction. Hvorslev already noted this effect. It is impossible that this many slippages are all reversible with rebound swell when the consolidation stresses are reduced in test B. The consolidation-rebound modified structure at B must be different, perhaps very much so, from the simple consolidation modified structure at A.

Consider now the practical case of an undisturbed soil with always-possible significant brittle cementing, or other bonding, between its particles. It then would have a true tensile strength in terms of effective stresses. What remains of this strength at failure must be part of "origin cohesion" and be included in c_e. It also seems likely that the strains of overconsolidation will irreversibly destroy some of these bonds that would otherwise not be destroyed, and then the total c_e at B will be less than at A (Fig. 1). Thus the assumption of equal c_e at equal void ratio would be in error and lead to values of c_e that are too low and of φ_e that are too high. Mencl (11) has made the same observation. It is known that determinations of φ_e for some undisturbed clays yield greater values than after remolding—which may, in part, be due to the above effect (12,13). Another possibility is discussed below.

Also implicit in the component determination illustrated by Fig. 1(*b*) is the assumption that both specimens fail on the plane of common tangency, which has the same inclination ($\alpha = 45 + \varphi_e/2$) in both specimens. But, as illustrated in Fig. 1(*c*), specimen A may fail along a plane with a different inclination than B. If ($\alpha_A \neq 45 + \varphi_e/2 = \alpha_B$), then the use of the common tangent gives too-high values of φ_e. On the other hand, if ($\alpha_A = 45 + \varphi_e/2 \neq \alpha_B$), then one obtains too-low values of φ_e. It seems likely that the less-disturbed specimen, A, will retain more of any natural brittle and/or anisotropic structure and thus $|\alpha_A - (45 + \varphi_e/2)| > |\alpha_B - (45 + \varphi_e/2)|$, and φ_e will be too high.

Shear Strain[4]—One has only to observe the development of a failure plane in a compression test to realize that shear strain can produce a structural change in the soil. The slickensides observed in some natural clays are evidence of the permanence of such structural changes. Structural distortion accompanying shear strain causes many local and irreversible slippages between particles as the structure struggles to mobilize its internal resistance to the forces producing the distortion. Recent investigations indicate that shear strain causes the particles to assume a more parallel, or dispersed, structure with increasing strain (14,15,16).

Referring to Fig. 1(*c*), suppose one tried to determine the effective components at different values of strain. At strain below ϵ^* this would lead to the unplausible result of negative φ_e. At greater strain it might lead to negative c_e. Obviously, this is not a correct procedure. Yet to consider together in Fig. 1(*b*) the two failures, each at different strain, is seemingly correct—at least in Hvorslev's investigation. Why? It can only be so if the initial differences in structure due to the overconsolidation were erased by the structural changes of strain, so that at failure at the same void ratio the structures were essentially identical. Under what conditions might this occur?

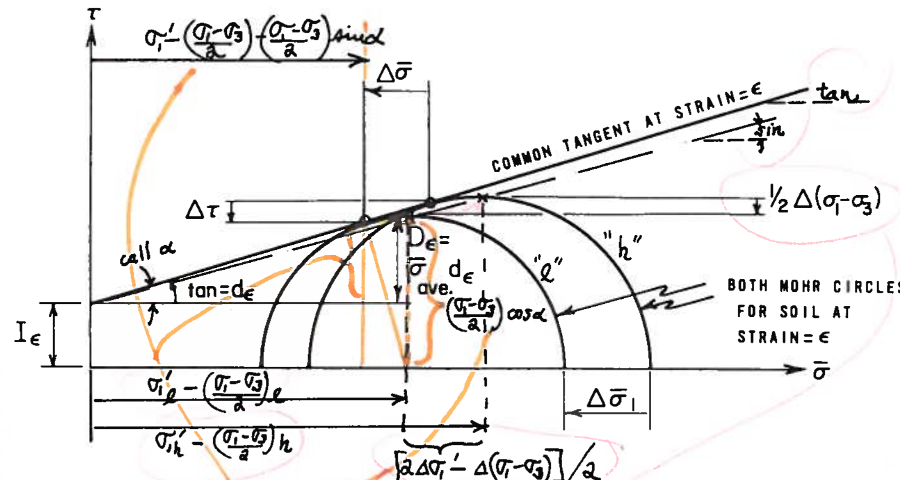
Hvorslev's Experiments — Hvorslev

[4] The writer does not differentiate between recoverable and nonrecoverable deformations or between elastic and plastic strains. The term "strain" as used here means total deformation divided by an appropriate undeformed dimension.

started with soils remolded at the liquid limit, so that brittle bonding or a highly flocculated initial structure were not possible. His use of direct and torsion shear machines tended to disperse the particles during consolidation in the

AXIAL STRAIN AT FAILURE, PER CENT.

Soil	Normally Consolidated	Overconsolidation ratio = 5
Vienna clay............	40	4
Little Belt clay.........	26	9



$$d_\epsilon = \tan\left\{\sin^{-1}\left[\frac{\Delta(\sigma_1-\sigma_3)}{2\Delta\bar\sigma_1 - \Delta(\sigma_1-\sigma_3)}\right]\right\} \quad \ldots\ldots\ldots\ldots (3)$$

$$I_\epsilon = \frac{(\sigma_1-\sigma_3) - [2\bar\sigma_1 - (\sigma_1-\sigma_3)]\sin(\tan^{-1}d_\epsilon)}{2\cos(\tan^{-1}d_\epsilon)} \quad \ldots\ldots\ldots (4)$$

also $= s\left[\dfrac{\sigma_{a_h} - \left(\frac{\Delta\sigma_a}{\Delta\sigma_1'/\sigma_h'}\right)}{2\sqrt{1-\left(\frac{\Delta\sigma_a}{\Delta\sigma_1'}\right)}}\right]$

$I_\epsilon = \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos\alpha - \left[\sigma_1' - \left(\frac{\sigma_1-\sigma_3}{2}\right) - \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin\alpha\right]\tan\alpha$

gives I_e

NOTE: $(\sigma_1-\sigma_3)$ AND $\bar\sigma_1$ FROM SAME, BUT EITHER, CIRCLE.

FIG. 2—Separation of Generalized Components, Illustrated for Plane of Envelope Tangency in a Triaxial Test.

same direction as the subsequent shear strains would tend to do and thus make a high degree of dispersion at failure all the more likely. Translating Hvorslev's failure strain from direct and torsion machines into the now more familiar compressive strain, and ignoring the possibility of greater strain in failure planes, the author obtained the following (17, p. 261):

It seems reasonable to conclude that failure strain was large in these tests, making it still more likely that pre-shear differences in structure were erased by the shear strain to failure.

Can we rely on this structural sequence to occur with other soils, such as undisturbed soils with a highly developed initial flocculated and/or anisotropic structure, or brittle soils which do not fail at high strain, or in other tests such as the triaxial where consolidation and shear dispersion directions are less likely to coincide? The author thinks not. In a recent paper Olson (18) indicates he may have similar thoughts. Even if we could rely on this for the failure condition, we definitely should not for the important strains less than failure. Any consideration of other strains must be preceded by more generalized definitions for the effective components.

basis, more generalized definitions for the effective components can now be written:

D_ϵ = *Dependent Component,* component of shear resistance mobilized on any plane and at any strain ϵ, which is dependent on effective stress on that plane according to the equation:

$$D_\epsilon = \left[\bar{\sigma}\left(\frac{\Delta\tau}{\Delta\bar{\sigma}}\right)\right]_{\Delta\bar{\sigma}\to 0} = \left[\bar{\sigma}\left(\frac{d\tau}{d\bar{\sigma}}\right)\right] = \bar{\sigma}d_\epsilon \ldots (1)$$

when, at ϵ, the shear resistance on that

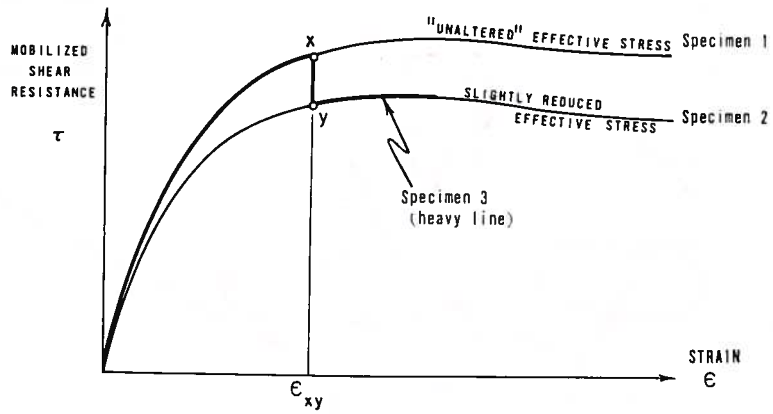


FIG. 3—Curve Hopping with One Specimen.

GENERALIZATION

Definitions:

Because structure is a function of stress history and strain, the components, or at least c_e, must also be. Furthermore, there is no reason to believe that c_e or φ_e (or both) are maximum values or that they both reach a maximum at the same strain, or that they are not indepenent during strain. Strain must be considered an independent variable. Then the concept of a failure is no longer of special significance, nor is the need to restrict the component separation to some future failure plane. Complete generalization requires that any plane can be considered. Using the intended meaning of the effective strength components as a

plane changes by $\Delta\tau$ due to $\Delta\bar{\sigma}$. Because there must be no change in soil structure, $\Delta\sigma$ must approach zero.[5]

I_ϵ = *Independent Component,* remaining component defined by:

$$I_\epsilon = \tau_\epsilon - D_\epsilon \ldots\ldots\ldots\ldots (2)$$

when τ_ϵ is the total shear resistance mobilized at ϵ on the plane considered.

In the incremental form necessary in experiments, the definitions are illustrated by Fig. 2 for the plane of envelope tangency (note similarity to Fig. 1(*b*)). The equations for this case are also given.

The author has taken the liberty of

[5] The certain, but perhaps minor, additional influence on $\Delta\tau$ of normal stress changes on the planes orthogonal to the plane considered is now ignored for simplicity. Therefore, this generalization must be considered incomplete.

changing the symbols for these generalized effective components. In previous publications he merely changed c_e to c_ϵ, and φ_e to φ_ϵ to emphasize they were functions of strain, ϵ. However, the physical meaning of these components may prove to differ considerably from the usual concepts of soil cohesion and friction. Therefore, continuing the philosophy of generalizing, they are merely called the components that are "dependent" and "independent" of effective stress, or D_ϵ and I_ϵ, respectively.

Experimental Approach:

The objective when measuring D_ϵ and I_ϵ is to obtain two effective stress conditions with the soil at the identically same structure and to do this at any strain. This seems impossible, and it is. Structure changes with effective stress and cannot be exactly the same, but perhaps it can be sufficiently so for a good approximation of the ideal. The measuring problem becomes one of achieving a relative maximum of effective stress change for a minimum of the inevitable structural changes.

Imagine that we can obtain two shear test specimens that are genuinely identical at the start of a shear test (zero strain in Fig. 3). We shear specimen 1 by some test method and obtain the curve shown. Then just before shearing specimen 2 we impose a slight change in the starting effective stresses. We then shear with the same test method and obtain the other stress-strain curve at all points of which effective stress is slightly different from 1. At any strain chosen we have two mobilized shear resistance values and two effective conditions and can therefore separate D_ϵ and I_ϵ, as shown in Fig. 2 for the plane of envelope tangency.

How great must the slight change be? The answer is the minimum necessary to separate the two circles in Fig. 2, so that a tangent can be determined with sufficient accuracy.

Curve-Hopping and Use of One Specimen:

The great practical difficulty in a component measurement method such as that just suggested (Fig. 3) is that it is often impossible to obtain two or more specimens that are sufficiently identical. Consider now a different technique with specimen 3. The small change in the initial effective stress condition of specimen 2 could also have been imposed at any point during strain (assuming it could be imposed without change of strain) such as x at ϵ_{xy}. If the subsequent shear resistance of specimen 3 at ϵ_{xy} is the same as that for specimen 2 (point y), then only specimen 3 is needed for component separation at ϵ_{xy}.

The only possible difference in structure between points x and y is due to the imposed decrease in effective stress which in turn results in a decrease in the shear resistance at ϵ_{xy}. The author's experience is that an adequate "hop" between curves results in a void ratio increase Δe of less than 1 part per 100. It has been a pleasant surprise to discover that it is possible to make several such hops during a strain-controlled test, and to make them in both the x-y and y-x directions, without greatly changing the subsequent position of the stress-strain curves (19). This curve-hopping technique is then at least approximately reversible with continuing strain. Either very little structural change occurs with each hop or the shear resistance effects of any change are almost recoverable with continued strain.

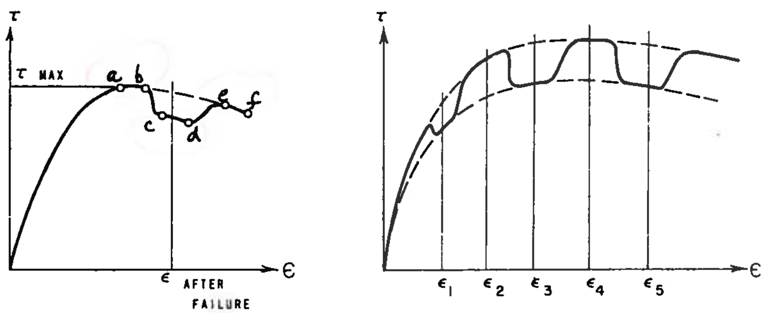
It is the author's opinion that the 1-specimen, curve-hopping separation of the generalized effective components results in the minimum structural difference between two conditions of effective stress that is presently possible with common soil mechanics techniques. It is at least an approximation of the minimum necessary.

The IDS Test:

The objective of this test is to separate the I and D effective components at any strain—hence the abbreviation IDS (CFS was used in previous publications because of previous cohesion and friction terminology). Curve-hopping can be used to the extent required. Two examples are given in Fig. 4. If the components are only desired for the after-failure shear resistance condition, then a hopping sequence such as a-b-c-d-e-f in Fig. 4(a), with interpolation between b-e, is sufficient for a separation at ϵ_{af}. If a continuous curve of the variation of D and I with strain is desired, then many hops may be required, as shown by the solid line in Fig. 4(b). After interpolation, again indicated by dashed lines, the component separation can be made at strains such as ϵ_1 to ϵ_5 and a curve fitted to illustrate their continuous variation with strain.

Variations in IDS Test—The concept of this test has proven to be applicable in many variations. It is the imposed change in effective stress that triggers the curve-hop. This change can be performed in any convenient manner as for example, two levels of $\bar{\sigma}_1$ wherein pore pressure is suitably controlled, or two levels of pore pressure or confining stress in drained tests, or two levels of constant volume in undrained tests with pore pressure measured. Wu et al used the test, in multi-specimen form, to study relaxation and creep. Bea (20), and Schmertmann and Hall (21) also studied time phenomena with 1-specimen tests using a short extrapolation to obtain pre-test conditions.

Equipment Requirements and Technique—To separate effective stress components on a particular plane at a particular strain it is only necessary to know mobilized shear resistance for two known effective stress conditions on that plane at that strain. The design of the equipment does not matter provided it can give this information. Presently, the triaxial is most suitable because of the relative ease of effective stress measurement. Any plane can be chosen. The direct shear apparatus is also suitable if the analysis is restricted to drained tests and the horizontal plane, as done by Landva (22). Theoretically even the undrained field vane test would be suitable to determine the average d component for the failure cylinder if somehow one could impose a controlled effective stress increment. The measurement precision required with any equipment can be estimated by review of the example given by the author in Table 5 of Ref. (19).



(a) IDS TEST AFTER CONVENTIONAL FAILURE (b) COMPLETE IDS TEST

FIG. 4—Examples of After-Failure and Complete One-Specimen IDS Tests.

The most difficult part of the 1-specimen technique is to ensure that the specimen has come to stress equilibrium after the imposition of an effective stress hop. The difficulty increases with decreasing permeability and increase in swelling tendency. The experimental techniques for the IDS test are intended to overcome this problem.

TABLE 1—AVAILABLE COMPARISONS OF tan φ_e AND d_{max} = max d_ϵ.

Soil	tan φ_e	d_{max}	Failure Criterion	Notes	Source
Remolded kaolinite..........	0.51		drained test $(\sigma_1 - \sigma_3)$ and $\bar{\sigma}_1/\bar{\sigma}_3$	constant volume direct shear tests	O'Neil (**25**)
	0.44 to 0.52		$\bar{\sigma}_1/\bar{\sigma}_3$	undrained triaxial	Schmertmann (**17**)
		0.51	$\bar{\sigma}_1/\bar{\sigma}_3$	constant volume IDS after failure	
		0.49	$\bar{\sigma}_1/\bar{\sigma}_3$	constant $\bar{\sigma}_1$ IDS	
Glacial Lake clay, PI = 32 per cent					
Undisturbed.....	0.19		$(\sigma_1 - \sigma_3)$	undrained triaxial	Wu et al (**15**)
		0.18	$(\sigma_1 - \sigma_3)$	multi-specimen, constant volume IDS	
Remolded.......	0.42		$(\sigma_1 - \sigma_3)$ at 10 per cent axial strain	undrained triaxial	
		0.41		multi-specimen constant volume IDS	
Undisturbed Leda clay..........		0.25	$(\sigma_1 - \sigma_3)$ and $\bar{\sigma}_1/\bar{\sigma}_3$	constant $\bar{\sigma}_1$ IDS	Schmertmann (**19**)
	0.31		Indirectly inferred before failure		Crawford (**26**)

fixed to a specified strain nor are they necessarily maximum values. As shown in Fig. 1(c), it is likely that the NC and OC specimens fail at a different strain. Then to what strain should c_e and tan φ_e be assigned for comparison with I and d? At conventional failure the sum of both components is a maximum, but it is very likely that both are not at their maximum values, so one cannot compare with both I_{max} and d_{max}. These comparison difficulties are illustrated by the author (17). A further complication is that Hvorslev used drained tests, and his work does not specify whether to use $(\sigma_1 - \sigma_3)$ or $(\bar{\sigma}_1/\bar{\sigma}_3)$ failure in undrained tests. It has become common to use $(\sigma_1 - \sigma_3)_{max}$ but the structural condition of the soil is probably more unique at $(\bar{\sigma}_1/\bar{\sigma}_3)_{max}$ (23,18 24).

COMPARISON BETWEEN HVORSLEV AND GENERALIZED COMPONENTS

In this paper the author attempts to show that the effective shear resistance components, D_ϵ and I_ϵ, can logically be considered a generalization of the effective strength components originally proposed by Hvorslev. If true, then at failure one should expect to be able to demonstrate that $I = c_e$ and $d = \tan \varphi_e$. But the comparison is more difficult because the c_e and φ_e components are not

Table 1 lists the presently available

comparisons of tan φ_e and d_{max}. This comparison was chosen as most likely to show agreement. IDS tests indicate it is very likely that I is past its maximum and decreasing at the strain of maximum shear resistance, which is also the strain of $(I + D)_{max}$. Therefore if $\bar{\sigma}$ is not changing too rapidly with strain, this is also approximately the strain of d_{max}. Maximum shear resistance is also the point for the tan φ_e determination and therefore should usually be in approximate agreement with d_{max}. The data presented generally support this argument. Unfortunately the list is brief and more comparisons are certainly needed.

Wu (27) offers some additional, though indirect, evidence. Using data from four remolded clays found in the literature, he plotted the variation of d_{max} with plasticity index and found agreement with a published tan φ_e versus PI curve averaged from many other remolded clays. Perhaps significantly, a similar comparison using undisturbed clays showed poor agreement, with d_{max} much less than tan φ_e. As discussed here, the determination of φ_e for undisturbed clays can involve serious errors with resulting too-high φ_e values.

CONCLUSION

The author recommends that the D_e and I_e shear resistance components be considered as possible generalizations of the Hvorslev effective strength components. If true, their determination is greatly simplified by the various forms of the IDS test. However, regardless of the merit of this argument, it is necessary for investigators to keep in mind the possibility that equal void ratio may not mean equal soil structure (cohesion) when determining the effective strength components for the conditions in their tests.

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(26) C. W. Crawford, "The Influence of Strain on Shearing Resistance of Sensitive Clay," *Proceedings*, Am. Soc. Testing Mats., Vol. 61, 1961.
(27) T. H. Wu, Discussion, Am. Soc. Civil Engrs. *Journal Soil Mechanics and Foundations Division*, July, 1963, p. 230.

DISCUSSION

ROBERT L. KONDNER[1]—The review of the various aspects of the strength and resistance of cohesive soils under applied stresses given by Schmertmann is a very interesting study in contrasts. Although the writer agrees with many of the points reviewed by Schmertmann, there are many points of disagreement.

Although he uses several parameters such as D_ϵ and I_ϵ or the previously used parameters, c_e or ϕ_e, in considering shearing resistance, Schmertmann is actually dealing with the stress-strain aspect of the stress-strain-time response of cohesive soils. In studying material behavior it is important, if possible, to express the response in terms of the fundamental variables under consideration, which in this case are stress, strain, and time, or in terms of parameters that are directly defined or calculated from them rather than artificial or indirect parameters. Schmertmann is attempting to study stress-strain-time behavior using the techniques or methods of representation used for studies of ultimate strength or failure stresses. Such techniques, as used by Schmertmann, are not compatible with material behavior at strains below failure.

The most widely used formulations of ultimate or failure strength of soils have their basis in the modified Mohr-Coulomb criterion expressed as a failure envelope in a two-dimensional stress space. This failure criterion concept does not refer to or correlate with the strain space; that is, it does not provide a relationship which specifies the coaxiality or orientation of the stress tensor relative to the strain tensor; nor does it provide a basis for an equational relation between them. The so called "cohesion" and "angle of internal friction" are fictitious soil properties which in reality are simply expedient parameters that have been used to approximate the representation of the failure envelope. It is indeed unfortunate that the names "cohesion" and "internal friction" were ever given to these parameters which are an "intercept" and a "slope," respectively, of an empirical curve fit. This has been recognized by some people for a long time and has been ably expressed by Lambe.[2]

In terms of failure stresses or shear strength in the plane of failure at the time of failure, for use in a form of limit or ultimate analysis, these two empirical parameters are useful engineering indices. However, there is no theoretical basis for extending the failure envelope concept to various stress-strain states below failure because strains at such stress states are not unique but are functions of the history or loading path of the soil up to the stress state as well as the nonlinear viscoelastic nature of the soil. Schmertmann seems to indicate a vague awareness of this at various points in his paper but then proceeds to ignore it by perpetuating the use of artificial parameters in introducing the quantities, D_ϵ and I_ϵ. The fact that there is a soil resistance to the applied loads or deformations is beyond question, but to divide this resistance into various parts with associated mechanisms of behavior is purely speculative and subject to considerable question. In addition, it would be highly unlikely for a number of investigators to agree on basic definitions and importance of any parts of such a division. The use of artificial parameters instead of fundamental variables can be very misleading in attempting to explain the stress-strain-time response of soils and, hence, is not to be recommended. This does not mean that all such parameters have no use as engineering indices for particular situations. In the future it might well be advisable for Schmertmann to consider stress-strain-time response of soils in terms of the fundamental quantities under consideration, namely, to express his results in terms of stress, strain, and time directly, in addition to the path and history variables.

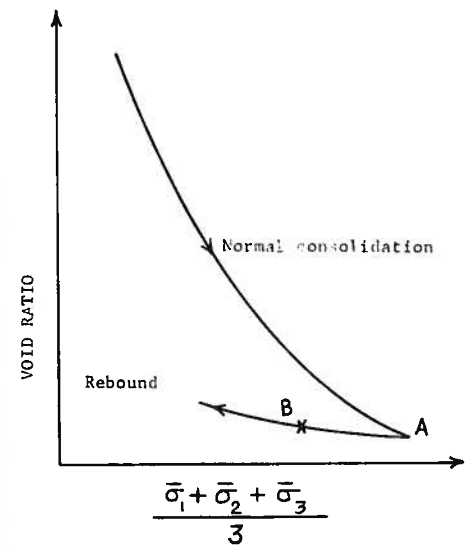

FIG. 5—Void Ratio-Pressure Relation.

The writer agrees with Schmertmann's contention that, in general, void ratio alone is not sufficient to describe the "state" of a soil. The term "state" may contain a variety of effects including such items as path dependence, history, and structure. The CFS or IDS test developed by Schmertmann is certainly an ingenious test and may prove to be a very valuable experimental tool for the soil mechanician. This may be particularly true in light of the difficulty of trying to obtain a number of genuinely identical soil specimens. However, the manner in which the test is conducted using one specimen with the curve-hopping technique is such that the instantaneous effective mean hydrostatic stress is continually changing; hence, one is continually operating on a portion of the recompression branch of the void ratio-pressure relation. This is shown qualitatively in Fig. 5, in which the test specimen is hydrostatically consolidated to point A. During the CFS curve-hopping test, the effective hydrostatic pressure decreases, and one is operating approximately along the recompression path AB. This is a consideration of significant importance. Both in the present paper and in other papers dealing with the CFS test, Schmertmann has indicated that it was possible to obtain approximately the same stress-strain curves for the single specimen with curve hopping as he obtained using a number of individual test specimens. All of these specimens are initially hydrostatically consolidated to the same void ratio as illustrated by point A of Fig. 5. The implication of the validity of the curve-hopping technique is an implication of the validity of the principle of superposition of effects, that is, linearity of effects. It is important to realize in making the substitution of one specimen for a number of specimens that all are related or associated with the vicinity of a single region of the e-p space

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[2] T. W. Lambe, "The Engineering Behavior of Compacted Clay," *Proceedings*, Am. Soc. Civil Engrs., Paper No. 1655, Vol. 84, No. SM2, 1958, p. 1655–20.

such as the region *AB* of Fig. 5. For such a situation, the overconsolidated nature of the soil for the recompression branch would indicate that in general the test is being completely conducted on a portion of an unloading curve with strain-hardened effects built into the soil structure; and, hence, one should expect a quasi-linear behavior. It is well known that the general principle of superposition does not hold for soils.[3] Indications are that the curve-hopping technique is valid only for small variations in the loading history of a soil. Thus, one must ascertain the effects of "slight" changes in the stress and deformation paths both before and during the test. Although Schmertmann states that void ratio alone is not sufficient to describe structural effects in soil response, he uses it as an indication of structural change and notes that the void ratio variation is less than one per cent. Thus, the extent of the general applicability of the CFS test, and the curve-hopping technique remains to be shown. Of equal importance is the form in which the test results are presented and the possible interpretations that are applied to these results.

JOHN H. SCHMERTMANN (*author's closure*)—Mr. Kondner's discussion does not concern itself with the main thesis of this paper, namely, that the *I* and *D* components can be considered generalizations of the Hvorslev effective components. Presumably he has no opinion about, or agrees with, this thesis. He does question in some detail the significance of the *I* and *D* components and the test technique to determine them. By implication he is also questioning the Hvorslev effective components.

For the purpose of efficient discussion I take the liberty of offering this concise restatement of Kondner's criticisms: (1) My omission of consideration of time, which is an important variable. (2) My perpetuation of cohesion and friction philosophy, which he considers to be nonfundamental. (3) In his opinion the use of Mohr circles for separation of components at strains less than failure is theoretically unjustified. (4) He sees this paper as implying the validity of superposition in soils, and he disagrees. I shall discuss these in this order.

1. One cannot but agree that time is an important independent variable. However, to simplify the main thesis of the paper I noted that methods of testing were to be considered constant. This was intended to be interpreted in the broad sense of including sample storage time, strain rate or rate of stress application, and any other time-dependent effect. I believe that a useful experimental approach toward a better understanding of the mechanism of shear resistance in soils is to consider stress-strain, strain-time and stress-time individually with the time, stress, or strain (respectively) held constant. This paper deals with stress-strain.

The generalized components can be, and have been, evaluated as functions of various time effects. Already published examples are Bea's (**20**)[4] study of creep (strain-time), the studies of creep and relaxation (stress-time) by Wu *et al.* (**15**), and Schmertmann and Hall's (**21**) study of creep and rate-of-strain (in closure).

2. Kondner seems to have missed the essential difference between the "intercept" and "slope" parameters obtained from the empirical straight line fit to approximate the Mohr failure envelop over the range of stress interest, and the generalized effective stress parameters I_ϵ and D_ϵ. With the *IDS* test the soil structure remains comparatively much more constant per unit change in effective stress. Parameters *I* and *D* are measures of a soil's shear resistance sensitivity at a given structure to a probing, seemingly nondestructive change of effective stress. This must be contrasted to the ordinary "cohesion" and "angle of internal friction" terms which represent the strength parameters from tests usually encompassing a large range in effective stress, with the different tests involving different failure modes, dilatancy behavior, failure strains, and void ratio at failure—in short, greatly different structure. The I_ϵ and D_ϵ symbols are used to help avoid missing this point.

It is clear from the history of soil mechanics that a great surge in our understanding of the engineering behavior of soils occurred with Terzaghi's laboratory and field demonstrations of the importance of effective stress. It is also clear that because soil deformation results primarily from sliding between particles the shear resistance of soil is fundamental to all shear and consolidation problems. Consequently, it is my opinion that the shear resistance sensitivity of a given soil structure to a change in effective stress, and the change of this sensitivity with strain, are of fundamental importance. I developed the *IDS* test as a practical means of determining this sensitivity.

3. The Mohr stress circle is simply a graphical representation of the distribution of shear and normal stress on any plane in a two-dimensional stress field, or where two of the principle stresses have the same magnitude. Static equilibrium is assumed—which is approximately true at all times with conventional rates of strain. The less the strain the more closely it is true because of reduced creep effects. At any strain, such as a strain less than failure, the two Mohr circles obtained from an *IDS* test permit a simple determination of how the normal stress and shear resistance on any plane changed in response to an imposed change in the effective stress conditions. The *I* and *D* components express this change, quantitatively, for that (any) plane.

The theoretical justification for the above use of the Mohr circles is as sound, and perhaps more so because of reduced creep, as that for their use at a defined "failure" condition. Confusion due to concern with stress history and time effects is unnecessary, because the generalized components are determined for the condition of the soil as found at the instant of the imposed effective stress change. As mentioned, stress history and time effects can be and have been studied independently.

The common procedure[5, 6] of indicating the progress of a triaxial test by means of stress paths assumes the validity of the use of the Mohr stress circle representation throughout the test. To my knowledge, this assumption has not been questioned previously in written discussion. This is a good assumption, with a degree of validity much greater than most assumptions made in soils engineering.

4. The experimental success of the curve-hopping procedure is a fact which the reader can verify (**17**). However, from this the author makes no implication of the validity of superposition or linear elasticity in soils. This implication is made by Kondner and he then argues against it.

Without here supporting the possible applicability of superposition, Kondner's

[3] K. Terzaghi, *Theoretical Soil Mechanics*, John Wiley and Sons, New York, 1943, pp. 394–395.

[4] The boldface numbers in parentheses refer to the list of references appended to the paper.

[5] A. Casagrande, and S. D. Wilson, "Prestress Induced in Consolidated-Quick Triaxial Tests," *Proceedings* of the Third International Conference on Soil Mechanics and Foundation Engineering, Switzerland, 1953, Vol. 1, p. 106, 1953.

[6] T. W. Lambe, "Methods of Estimating Settlement," *preprint* submitted to the American Society of Civil Engineers Conference on The Design of Foundations for the Control on Settlements, Session 1, Evanston, Ill., June, 1964.

argument against it is weakened by his choice of reference. On the pages cited Terzahgi said essentially (in 1942) that superposition is not valid for the case of layered systems in which each layer has a different modulus. Curtis and Richart[7] showed in 1955 that it was valid for this case.

Referring to the success of curve hopping, I stated "Either very little structural change occurs with each hop or the shear resistance effects of any change are almost recoverable with continued strain." If the latter (partial collapse of soil structure, recovery with strain, partial collapse, etc.) explanation is correct, then the soil's seemingly elastic behavior is only a superficial observation.

Mention of void ratio change during a single curve hop serves two purposes. The small magnitude of this change is a necessary, though of course not a sufficient, condition for indicating a small change in structure. However, it should be remembered that the main thesis of the paper is the relationship between the Hvorslev and *IDS* effective components and that the Hvorslev components are determined with no change in void ratio. To compare with the *IDS* components, wherein a small change in void ratio is permitted, it was considered useful to indicate just how small this change usually is.

[7] A. J. Curtis and F. E. Richart, Jr., "Photoelastic Analogy for Nonhomogeneous Foundations," *Transactions*, American Society of Civil Engineers, Vol. 120, 1955, p. 35.

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